Design of Lossless Junction, Given One Row in Its Scattering Matrix

It is desirable occasionally to find a lossless n-port junction which has a scattering matrix with a specified row. In some cases, it may be sufficient to know that such a junction exists. An example of how such a junction might be used is given later.

In order to illustrate how the desired junction can be designed and the corresponding scattering matrix found, a specified case is considered. The generalization to any other case is obvious. Since the numbering of the ports is arbitrary, it is assumed that the first row is given. For the specific case considered, the elements in the given row are

$$S_{1k} = S_{1k}\underline{/\theta_{1k}}, \qquad (k = 1, 2, 3, 4)$$
$$S_{15} = 0.$$

The equivalent circuit for the desired junction is shown in Fig. 1.

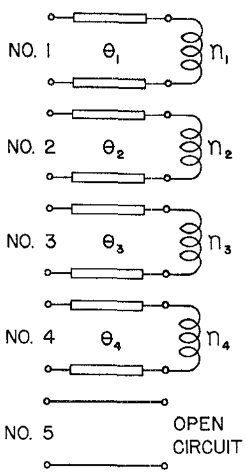

Fig. 1. An equivalent circuit for the desired junction.

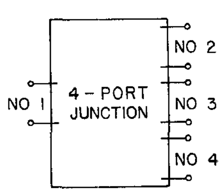

Fig. 2. An arbitrary junction.

The equivalent circuit consists of sections of transmission line and a multiwinding ideal transformer. The sections of transmission line produce the phase delays indicated, with

$$\theta_1 = -\theta_{11}/2,$$
$$\theta_k = -\theta_{1k} - \theta_1.$$

The turns ratios for the windings of the ideal transformer are

$$\frac{n_k}{n_1} = \frac{S_{1k}}{1 + S_{11}} \qquad (k = 2, 3, 4) \cdot$$

The writer will not bore the reader with the tedious details of verifying that the circuit shown in Fig. 1 does have the desired characteristics.

Next, consider the junction shown in Fig. 2. The junction is not necessarily lossless. Let the scattering matrix of this junction be denoted by (J). It is assumed that power is fed into port No. 1 and that the remaining ports are terminated in matched loads. The efficiency of the junction under these conditions is

$$\eta = (J_{12}^2 + J_{13}^2 + J_{14}^2)/(1 - J_{11}^2).$$

It is possible to design a lossless 4-port junction, which is called the *collector junction*, which combines the outputs of ports 2, 3, and 4 into a single output port. Let the scattering matrix of the collector junction be denoted by (C). The output port is designated the No. 1 (C) port. When the two junctions are connected, as shown in Fig. 3, the combination is a 2-port junction.

Let the elements in the first row of (C) have the values

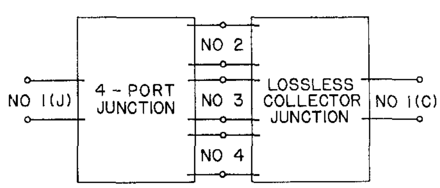

Fig. 3. Combination of arbitrary junction and collector junction.

$$C_{11} = 0,$$
$$C_{1k} = J_{1k}^*/\sqrt{J_{12}^2 + J_{13}^2 + J_{14}^2}, \qquad (k = 2, 3, 4)$$

where the asterisk denotes the conjugate. The equivalent circuit for the collector junction is the same as the one shown in Fig. 1, omitting port No. 5. Now ports 2, 3, and 4 of the original junction are terminated in matched loads, and the efficiency of the 2-port junction has the value η given above. (It might appear from the last remark that all C_{kk} must be zero, but this is not true.) The voltage transmission coefficient for the 2-port network is

$$T = \sqrt{\eta}.$$

If port No. 1 (C) is terminated in a matched load, the voltage reflection coefficient at port No. 1 (J) is

$$\Gamma = J_{11}.$$

These remarks might be used to consider this question: *Is it theoretically possible to build a lossless antenna which collects all the incident power?* However, this question is too complex to be treated here.

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Manuscript received November 1, 1965.

A Method of Calculating the Attenuation Constants of the Unwanted Modes in Mode Filters Using Resistive Sheets

Mode filters, which are one of the most important components in overmoded waveguides, give no attenuation to the main mode and large attenuations to some unwanted modes. This correspondence deals with mode filters using resistive sheets.

In these mode filters, the resistive sheets are mounted in a hollow waveguide so that they are always perpendicular to the electric field of the main mode, while the electric fields of the unwanted modes have tangential components along the resistive sheets.

In our discussion, the following three assumptions are postulated.

1) The thickness of the resistive sheet is much smaller than the skin depth, and its impedance is pure resistance. (In the millimeter wave region, a thin metal film deposited on a thin dielectric base is desired.)

2) The square resistance ($R_\square$) of the resistive sheet is large enough, and it does not disturb the field pattern of the waves in the hollow waveguide when it is inserted into the waveguide.

3) The attenuation of the unwanted modes is due to the heat loss in the resistive sheet according to the conduction current $J = \sigma E$ (σ = conductivity of the resistive material, E = tangential electric field along the resistive sheet).

Under the above assumptions, if the resistive sheets are inserted into a straight waveguide and have the same cross section along the guide axis, the attenuation constant of any unwanted mode may be given approximately by

$$\alpha = 8.686 \frac{1}{2P} \int\!\!\int \frac{1}{2\sigma} \, |J|^2 dS \quad \text{dB/m}$$

P = transmission power of any unwanted mode

where the surface integral is taken over the transverse cross section of the resistive sheets of thickness t, and its value is equal to the power losses in the resistive sheets per unit length along the axial direction.

Manuscript received November 5, 1965.